MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2017/2018

DEM5028 – ENGINEERING MATHEMATICS 2

11 OCTOBER 2017
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **4** pages (**2** pages with **4** questions and **2** pages of appendix.
2. Answer **ALL** questions. All necessary working steps must be shown.
3. Write all your answers in the answer booklet provided.

QUESTION 1 [25 MARKS]

a. Using suitable integration method, evaluate the following integrals.

i) $\int 4x^3\sqrt{x^4+12}dx$

[4 marks]

ii) $\int 3xe^{-x}dx$

[5 marks]

iii) $\int_0^1 \frac{dx}{14+x^2}$

[3 marks]

b. Find the area of the enclosed region bounded by $4y = x + y^2$ and $x = y^2 - 2y$.

[5 marks]

c. Find the solution of the differential equation $\frac{dy}{dx} = x(1-4y)$.

[8 marks]

QUESTION 2 [25 MARKS]

a. Test the series for convergence or divergence. (Must state the test used)

i) $\sum_{n=1}^{\infty} \frac{1}{5+2^n}$

[4 marks]

ii) $\sum_{p=1}^{\infty} p^2 e^{-p}$

[5 marks]

b. Consider the power series $\sum_{n=1}^{\infty} \frac{n(x-8)^n}{2^n}$

i) Find the radius of convergence.

[4 marks]

ii) Find the interval of convergence.

[4 marks]

c. Find up to second Taylor series $(n = 2)$ for $f(x) = xe^x$ at $a = 3$ .State the final answer in sigma notation.

[6 marks]

d. Find the first two terms (n=2) of Maclaurin Series for the same $f(x)$ in Question 2 c.

[2 marks]

Continued...

QUESTION 3 [25 MARKS]

a. Given point A is (2,1,−3) and B is (1,4,3) Find:

i) Position vector of $\vec{A}$

[2 marks]

ii) $\vec{A} \cdot \vec{B}$

[2 marks]

iii) Evaluate $\vec{B} \times \vec{A}$

[4 marks]

b. Find the value of t if vector $\vec{A} = < t+1, t+2, t >$ and $\vec{B} = < t, t-1, t-1 >$ are orthogonal.

[5 marks]

c. Find the equation of the plane that contains the line $x = 1+2t, y = 2+t, z = 1+4t$ and the line $x = 3-2t, y = 5-3t$ and $z = -t$.

[6 marks]

d. Find the vector equation, parametric equation and symmetric equation of the line that through $< 2,5,3 >$ and parallel with vector $2\hat{i} - \hat{j} + 2\hat{k}$.

[6 marks]

QUESTION 4 [25 MARKS]

a. For $f(x, y, z) = x\cos z + x^2 y^3 e^z$, find all of first derivatives.

[6 marks]

b. Given $z = x^2 y - y^2$, $x = t^2$ and $y = 2t$. Find $\dfrac{\partial z}{\partial t}$.

[5 marks]

c. Find all the critical points for $f(x, y) = -12x^3 + 2y^2 + 18x - 6y$.

[4 marks]

d. Determine whether the critical points for $f(x, y) = \dfrac{x^2}{2} + y^3 - 3xy - 4x + 2$ are local maximums, local minimums or saddle points.

[10 marks]

End of Page.

APPENDIX I: Formulae

Integration of common functions

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} dx = \ln|x| + C \qquad \int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x\, dx = \sin x + C \qquad \int \sec^2 x\, dx = \tan x + C \qquad \int \sec x \tan x dx = \sec x + C$$

$$\int \csc^2 x dx = -\cot x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left([f(x)]^2 - [g(x)]^2\right) dx$$

$$V = \int_c^d \pi\left([w(y)]^2 - [v(y)]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$

$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Linear Differential Equations:

$$\frac{dy}{dx} + p(x)y = q(x);\ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \qquad \text{where } \mu = e^{\int p(x) dx}$$

Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
p-series	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ If $0 < c < \infty$, then both series converge or both diverge.
Alternating Series Test	If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \qquad b_n > 0$ Satisfies : i. $b_{n+1} \leq b_n$ for all n ii. $\lim_{n\to\infty} b_n = 0$ then the series is convergent
Ratio Test	Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$ a. Series converges absolutely if $L < 1$ b. Series diverges if $L > 1$ or $L = \infty$ c. No conclusion if $L = 1$

Taylor Polynomials

$$T_n = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

Vector

The length of the vector $\mathbf{a} = < a_1, a_2, a_3 >$ is $|a| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$.

If θ is the angle between the vector **a** and **b**, then $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Cross Product

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

Equation of Line

Vector equation: $r = r_0 + tv$ Parametric equation: $x=x_0+at$, $y=y_0+bt$, $z=z_0+ct$

Equation of Plane $< a, b, c >< x - x_0, y - y_0, z - z_0 >= 0$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a, b) is a critical point of f]. Let

$$D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.

b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum

c. If $D < 0$, then $f(a, b)$ is a saddle point.